THINGS

of science

SYMMETRY

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SYMMETRY

The perfectly curved arch, a spoon, a snowflake, the butterfly, a Saturn rocket and man, all have one thing in common. They are all symmetrical, at least on the outside.

If you observe each of them carefully, you will see that their shapes are balanced and in pleasing proportions, and that their contours are arranged harmoniously. Their right and left sides look alike.

Nature abounds in examples of symmetry. Man from his earliest days was aware of symmetry in nature and expressed this in various ways, in art, architecture and everyday utensils. Because symmetrical objects are generally pleasing to the eye, symmetry is often associated with the sense of beauty.

Symmetry appears in various forms and has been classified into several types. With the materials in this unit, you will be able to observe some of the characteristics of symmetry and learn how to recognize them.

First examine your materials.

STYROFOAM BALLS—Nine

ZINNIA SEEDS—One packet

SALT—One packet

FINE COTTON-COATED WIRE—

Seven $4\frac{1}{2}$-inch pieces

STIFF PAPER—Four 3x5-inch pieces

LIGHTWEIGHT PAPER—One 5x12-inch folded sheet

WHAT IS SYMMETRY?

Symmetry can be analyzed geometrically. A figure or object is said to be symmetrical if it has balance with reference to a point, line or plane.

An object or figure that is not symmetrical is called asymmetrical.

POINT SYMMETRY

Experiment 1. Take one of your 3x5-inch cards and find the center of each of the sides. Draw lines along the center from one side to the other. Make a dot at the point at which the lines intersect and label it O. Now draw another straight line through O starting at any point along one edge to the opposite edge (Fig. 1). Measure the lines. Is each line divided into equal parts by the point?

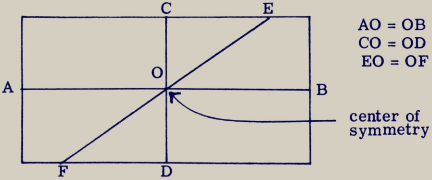


Fig. 1

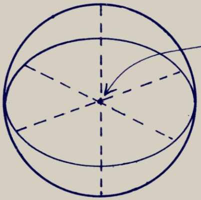


Fig. 2

Draw other lines through the point. Are they also bisected (divided into two equal parts)?

A figure is said to have point symmetry if a point can be found such that all lines passing through that point and terminated by the figure are bisected by the point. Such a point is called the center of symmetry.

A solid object such as a sphere also has point symmetry (Fig. 2). Can you tell why?

Experiment 2. Can you show that an equilateral (all three sides equal) triangle has point symmetry?

Experiment 3. Write out the alphabet in capital letters. In which of the letters do you find examples of point symmetry?

LINE SYMMETRY

Experiment 4. Fold the stiff paper used in Experiment 1 along the center with the blank side facing up. Does the fold line divide it into two equal parts? Draw lines across the paper perpendicular to the fold line. Does the line divide all the perpendiculars extending from edge to edge into equal parts (Fig. 3)?

A figure is said to have line symmetry or to be symmetric with respect to a line if such a line bisects all line segments which are perpendicular to it and which are terminated by the figure. Such a line is also called an axis of symmetry.

The paper or rectangular figure, as you can see, has line symmetry.

Experiment 5. Draw a circle. Does

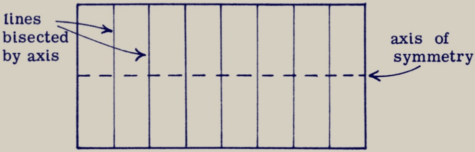


Fig. 3

it have line symmetry? Where is its axis of symmetry? Does it have more than one?

Experiment 6. Take one of your styrofoam balls and pass one of your fine wires directly through the center of the sphere. Can you show that the sphere has line symmetry? The line represented by the wire is the axis of symmetry of the sphere.

Experiment 7. Where is the axis of symmetry of the letter A? Of letters E and H?

Experiment 8. Many figures have more than one axis of symmetry.

Cut out an equilateral triangle from a half of one of your stiff papers.

Draw a line from a vertex to the opposite base. This line represents an axis of symmetry. Can you show why? Find its two other axes of symmetry and draw lines to represent them. Now you can see that an equilateral triangle has three axes of symmetry.

Note that the lines all meet at one point. This point is the center of symmetry.

Some flowers have three axes of symmetry. Can you name them?

Experiment 9. Cut out a square from the other half of the paper. How many axes of symmetry does it have? You can discover this by folding the paper in various ways. Do you find four axes of symmetry? Can you find the center of symmetry?

Experiment 10. Cut a cross from the square (Fig. 4).

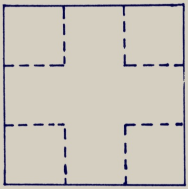


Fig. 4

How many axes of symmetry does this cross have?

Experiment 11. Can you show that a circle has an infinite number of axes of symmetry?

PLANE SYMMETRY

Experiment 12. Remove the wire and cut the styrofoam ball used in Experiment 6 in two.

If all line segments in a solid figure perpendicular to a plane and terminated

by the solid are bisected by the plane, the solid is said to have plane symmetry. The plane by which an object is divided into two symmetrical parts is called the plane of symmetry.

Can you show that a sphere has plane symmetry (Fig. 5)? Does it have more than one plane of symmetry?

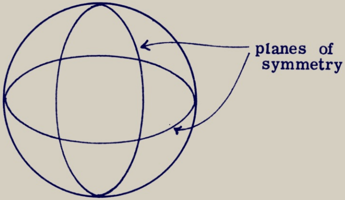


Fig. 5

Experiment 13. What other solid figures show plane symmetry? Where would the plane of symmetry be in a pyramid? Is there more than one?

TYPES OF SYMMETRY

There are several types of symmetry: universal, radial, bilateral, rotational and translational. A symmetrical object may have a single type or a combination of two or more types.

UNIVERSAL SYMMETRY

Experiment 14. Examine one of your styrofoam balls and note that it looks the same from whichever side you observe it. Take the halves of the styrofoam ball you cut in two in Experiment 12, and note that they are exactly equal. You have two symmetrical halves.

You would obtain two symmetrical sections no matter from which side you cut the styrofoam ball, as long as the plane passes through the exact center.

Such an object is said to have universal symmetry.

Any plane that passes through the center of an object having universal symmetry will divide the figure into halves that are symmetrical.

Among living organisms, a few of the lowest forms, such as the volvox and a few of the protozoa, have universal symmetry.

An object having universal symmetry also has plane symmetry (see Fig. 5). Does it also have point symmetry and line symmetry? See Experiments 1 and 6.

Glue the two halves of the styrofoam ball together again for use in a later experiment.

Experiment 15. Roll one of your stiff sheets of paper lengthwise into a cylinder and tape the edges together without overlapping them.

You now have a right circular cylinder. A line joining the centers of the bases is the principal axis of the cylinder (Fig. 6).

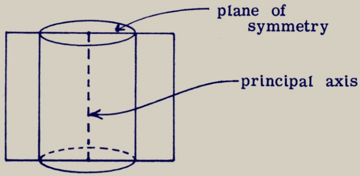


Fig. 6

Take another one of your cards and place it perpendicular to the base. If you passed this card down the cylinder along the principal axis, it would divide the cylinder into two symmetrical halves. Are there more planes of symmetry than one that will divide the cylinder into two equal parts? You will find there are a number of planes that may be passed through the principal axis that will divide the cylinder symmetrically.

This type of symmetry shown by the cylinder is called radial symmetry.

Radial symmetry exists in lower forms of animal life, such as the hydra and starfish. These animals, like a cylinder or an umbrella, are symmetrical around the principal axis of the body. Many flowers also have radial symmetry.

Experiment 16. You have observed that the cylinder has both plane and line symmetry. Does it also have point symmetry?

Experiment 17. Can you name other solid figures that have radial symmetry?

BILATERAL SYMMETRY

Experiment 18. Bend one of your 4½-inch wires to form the letter M. Draw a vertical line on a sheet of paper and

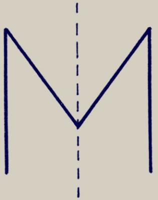


Fig. 7

place the center of the M on it (Fig. 7).

Note that the right and left sides look alike. They are equivalent. The letter M as you can see is symmetrical.

Place a small mirror on the line. You will see the letter M, one-half of which is on the paper and the other half its reflection in the mirror, or plane. One side of the M is the mirror image of the other. This is the symmetry of left and right and is called bilateral symmetry. It is the symmetry most frequently found in nature and is characteristic of all the more highly developed animal types, where each of the halves separated by a plane through the middle is a mirror image of the other. The symmetry operation in bilateral symmetry is called reflection in a plane.

Experiment 19. Open up the sheet of folded lightweight paper. Cut a 3x5-inch piece from it and then fold it in half crosswise. Unfold it and place a drop of ink on the center of the fold. Then press the halves together, smoothing the paper flat to spread the ink. Open up the paper. Do you have a design showing bilateral symmetry? Can you see from this that bilateral symmetry is reflection in a plane?

The word bilateral is derived from the Latin *bi* which means two and *lateral* meaning side. Bilateral symmetry thus refers to two-sided symmetry.

Figures having bilateral symmetry may be either two-dimensional, like the ink spot, or three-dimensional, like the styrofoam ball that you cut into two equal halves.

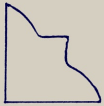
Experiment 20. Face a mirror and look at your reflection. Are you bilaterally symmetrical? Place the palms of your hands together. Your left and right hands are mirror images. You appear symmetrical on the outside, but the symmetry does not extend to all the organs within your body. Your skeletal structure, however, shows bilateral symmetry.

Can you recognize bilateral symmetry in various insects and animals?

Experiment 21. Look at your image from the side with the help of a second mirror. Are you symmetrical from the side? The human body has only one plane of symmetry.

Experiment 22. Cut a 3x3-inch piece from your thin sheet of paper and fold it in half one way and then in half again from the other direction. Draw the design

below (Fig. 8a) on the folded paper with the right angle at the fold. Open up the paper and you have a bilaterally symmetrical figure (Fig. 8b).


a.

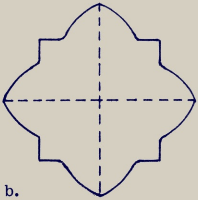

b.

Fig. 8

Turn the figure one-quarter turn (90°). Is it bilaterally symmetrical at this position also? Does the figure have more than two planes of symmetry?

Experiment 23. How many letters of the alphabet have bilateral symmetry? Do any of the letters have more than one plane of symmetry?

Experiment 24. Objects having universal and radial symmetry are usually bilaterally symmetrical also. Can you demonstrate this?

Can you recognize bilateral symmetry in a jet plane, automobile, square table, saucer, a bivalved shellfish, leaves, animals, some fruits and vegetables?

ROTATIONAL SYMMETRY

Experiment 25. Cut an equilateral triangle from a 2x3-inch section of your lightweight paper. Draw a line along one side to identify it. Place the triangle with the marked side at the bottom, then turn it so that the side to the right is at the bottom and then turn the triangle so that the third side is at the bottom (Fig. 9). Does the triangle appear the same in all three positions (disregarding the identification line)? The triangle has threefold rotational symmetry.

A figure having rotational symmetry appears identical in a certain number of positions, no matter what position you start with. Turn the triangle upside down and then turn it so that a different angle is at the bottom each time. Does the triangle again show three-fold symmetry?

Experiment 26. Repeat the experiment with a 2x2-inch square cut from the lightweight paper, marking one corner

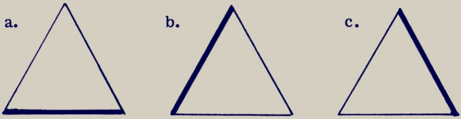


Fig. 9

with a dot. In how many positions does it appear the same? The square has fourfold rotational symmetry.

Does the pattern you cut out in Experiment 22 have rotational symmetry? In how many positions does it appear exactly the same?

Experiment 27. Draw a large letter U on a 3x3-inch sheet of your lightweight paper and cut it out. Take the square you used in Experiment 26 and rotate the U and the square at the same time into four different positions, making 90° turns (Fig. 10). Does the letter U appear the same in the four positions? You can see that U has bilateral symmetry (Fig. 10a), but not rotational symmetry, since it appears different with each change in position. The square has both symmetries.

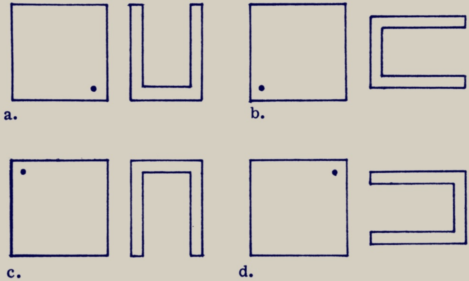


Fig. 10

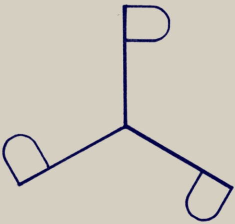


Fig. 11

Experiment 28. Most figures having rotational symmetry also have bilateral symmetry. Can you demonstrate this?

Experiment 29. Some figures, however, have rotational symmetry, but not bilateral symmetry.

Cut three 2¼-inch lengths from your wires and shape each into the letter P. Place the three wire-formed P's on a flat surface with the stems touching at the tip and equidistant apart (Fig. 11). Turn the figure around into three different positions at 120° intervals.

Does the figure have rotational symmetry? What about bilateral symmetry?

Can you draw other figures that show rotational but not bilateral symmetry?

Many flowers have rotational symmetry. What type of symmetry does a pine cone have? A wheel? A propeller?

DESIGN SYMMETRY

Experiment 30. Make three more P's with $2\frac{1}{4}$-inch pieces of wire. Place the six P's in a row. You have a series of repeated patterns. This type of symmetry is known as translational symmetry. Many examples of such repeated designs are seen in borders, pottery and bas reliefs. In architecture the repeated evenly spaced pillars of ancient temples show how translational symmetry may be used.

Shape the remaining wires into circles and place them between each of the P's at equal intervals. This is translational symmetry with reflection.

In nature, some plant shoots and leaf formations, as well as the centipede show translational symmetry. Can you think of other examples?

By making use of the various symmetry operations, you can make many interesting designs, such as repeated patterns of the same type in a total surface. Many wall papers and fabrics have such patterns.

Experiment 31. Fold one of your lightweight papers in half lengthwise and then in half again crosswise. Cut triangular pieces of various sizes along the folded edges and open the paper to its original shape. What types of symmetry do you

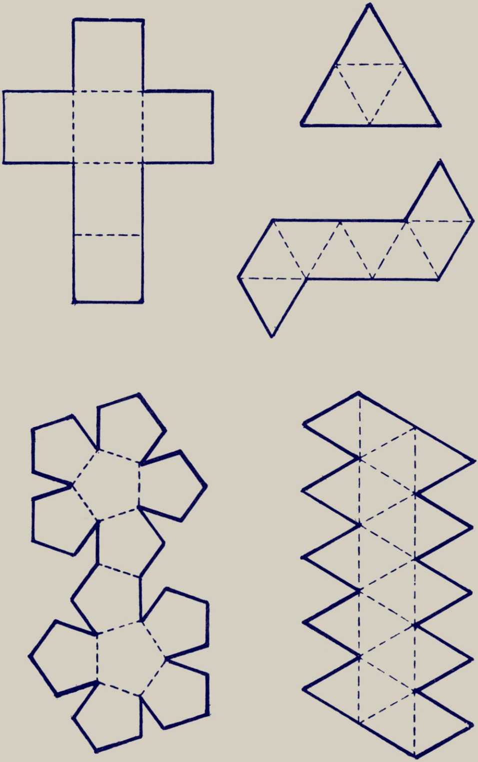


Fig. 12

observe? Find the axes of symmetry of your design.

Experiment 32. Draw the patterns in Fig. 12 onto large pieces of stiff paper. These are regular polyhedra, three-dimensional figures having four or more sides. Cut out the figures and fold them along the dotted lines, securing the edges with cellophane tape.

These five regular polyhedra are examples of solid figures having faces which are symmetrical plane figures. They are the only convex regular solids which can be made from symmetrical plane figures.

SYMMETRY IN CRYSTALS

One of the most fascinating examples of symmetry in nature is the symmetry of crystals. If a crystal is allowed to grow freely, it develops along straight lines, usually to form a polyhedron. It is bounded by a certain number of flat faces that meet at the edges and at the corners.

The characteristic shape of any crystal depends upon the crystal formation of the smallest unit of the substance, called the unit cell, and the way in which these unit cells, which are like tiny building blocks, are stacked.

Thus from the external symmetry of crystals, one can gain information about the arrangement of the unit cells in a substance.

Table salt, or sodium chloride, nearly always crystallizes in the form of cubes with faces which intersect at an angle of 90°. It can, however, also be made to crystallize as octahedral (eight-faced) crystals.

Experiment 33. Prepare a concentrated solution of table salt by mixing about a tablespoonful of water with the salt in your unit. Stir it thoroughly adding a little more water if necessary, but allow some of the salt to remain undissolved to insure a saturated solution. Pour the concentrated solution into a clean shallow dish, leaving any grains of salt behind. Allow the solution to remain undisturbed overnight. As the water evaporates, crystals of salt will be formed. Examine them. Are they cubical?

Some of the crystals may be rectangular rather than cubical since they can grow freely only upward and to the sides as they rest on the bottom of the dish. However, you can see that the edges are perpendicular to each other and the corners are precisely formed.

Experiment 34. Select a large, nearly round potato and with a sharp knife cut from it a cube two inches on each side. Measure the sides with a ruler and see that all edges are of equal length and that the angles are perfect right angles.

While it may seem to be a long distance from a potato to a crystal, we can study the simplest kind of symmetry which occurs in crystalline substances, known as simple cubic, by this means. Each point at the corner of the potato cube represents a position occupied by an atom or molecule. We can demonstrate this with the styrofoam balls.

Cut four of the wires into 12 1½-inch lengths. Insert the wires into eight of the styrofoam balls to form a cube to illustrate simple cubic symmetry (Fig. 13). The framework of cubic crystals is a lattice in three dimensions and is called a space lattice. These lattices can continue indefinitely in every direction, the atom or molecule at each corner being shared by the eight cubes that meet there.

Experiment 35. The atoms in a cubic unit cell may be so arranged that there are 8 atoms at the center of the edges (or one at each corner) and an atom at the center. This is known as a body centered cube.

Place the ninth styrofoam ball in the center of your cubic model to demonstrate a body centered cube.

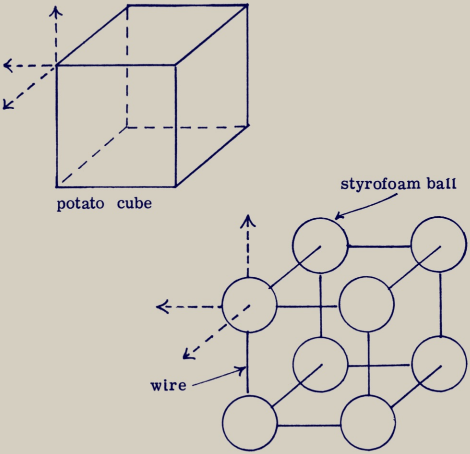


Fig. 13

Experiment 36. With a ball-point pen draw lines lightly across each face of the potato cube diagonally from corner to corner. Place a dot at the center where the two lines cross. If an atom lies in the center of the face of a cube, the structure is called a face-centered cubic. Part of each

face-centered atom is in each of the two cubes that meet at that face.

Can you illustrate a face-centered cube with the styrofoam cube?

Experiment 37. Mark the center of each edge of the potato cube with the ball-point pen. Draw lines from each point to those adjoining it, thus marking off a square on each face of the cube (Fig. 14).

Now gradually shave off the eight corners of the cube, keeping the blade of your knife at right angles to the sides of the cube, until you have reached the lines. Continue to shave the potato in the same manner. You will produce eight new faces (the original faces of the cube will disappear) and the new structure is an octahedron with eight triangular faces (Fig.

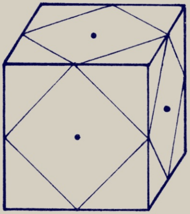


Fig. 14

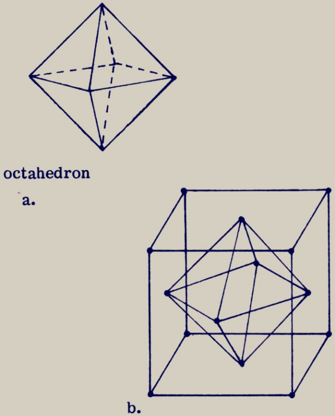


Fig. 15

15a). The face-centered atoms become the corners of the octahedron (Fig. 15b).

The relation between a cube and an octahedron accounts for the fact that some substances having face-centered cubic symmetry may crystallize as octahedrally shaped crystals.

Experiment 38. From another potato cut a 1½-inch cube. Pass one of your 4½-inch wires through the center of two opposite faces. If the wire does not go through easily, insert a needle first to

guide it. Turn the cube. Do you find that it looks the same for each quarter twist? The wire represents the axis of four-fold rotational symmetry. Each cube has three axes of four-fold symmetry. Can you find the other two? (Fig. 16a.)

Pass the wire through opposite edges. Rotate the cube and you will find that this axis is one of two-fold symmetry. Count the edges. You will find 12 edges or six pairs of edges which are opposite and parallel, giving six axes of two-fold symmetry. (Fig. 16b.)

Insert the wire through opposite corners. How many such pairs of corners are there? Rotate the cube. Do you have three-fold symmetry? How many axes of three-fold symmetry does a cube have? Fig. 16c.)

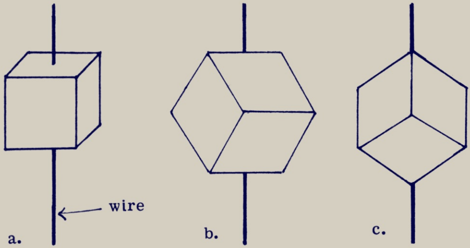


Fig. 16

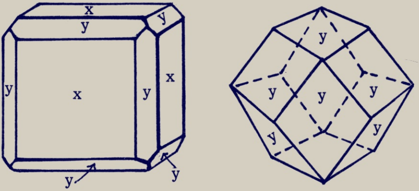


Fig. 17

Experiment 39. How many planes of symmetry do you find? Each plane must divide the cube into mirror images. Pass the plane through the edges as well as the sides.

Can you locate the center of symmetry of a cube using the styrofoam balls and wires?

Experiment 40. Cut another two-inch cube from a potato. Mark the faces x. Slice off the edges, a small amount at a time (Fig. 17). Mark these faces y.

Continue to cut away the edges. The x faces gradually decrease in size as the y faces become larger, finally leaving only the y faces, producing a symmetrical 12-faced figure. To keep track of the y faces, mark them as you slice them.

How many symmetry operations can you find in this figure?

Experiment 41. Draw arcs on one of the styrofoam balls (Fig. 18) to form symmetrical spherical triangles. Use a water color pen to color triangles A, B, C and A^1, B^1, C^1.

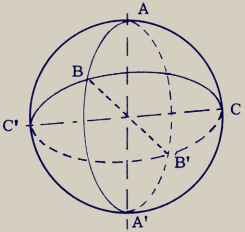


Fig. 18

Symmetrical spherical triangles are those in which the sides and angles of one are equal respectively to the sides and angles of the other, but arranged in reverse order.

Two symmetrical triangles are mutually equilateral and equiangular, but in general they cannot be made to coincide.

SYMMETRY IN PLANTS

Experiment 42. Plant several of your zinnia seeds in vermiculite or sandy soil in a pot about ⅛-inch deep.

As the seeds germinate note that the first leaves grow opposite each other and appear at the same time. This characteristic is usual with all dicotyledons. What type of symmetry is this?

After the seedlings have grown about two inches, plant them in the sunshine outdoors. As the plant grows, note that leaves appear in pairs symmetrically, but that alternate pairs are crosswise.

Does the stem of the zinnia show radial symmetry?

Experiment 43. Observe the flower when it blooms. What types of symmetry does the flower show?

Experiment 44. Are the leaves bilaterally symmetric?

Most symmetry in plants is imperfect, just as in humans.

Experiment 45. Observe the types of symmetry in various other plants and flowers.

Symmetry enters into all areas of our lives. As you have observed, the various types of symmetry are found in all fields of science—chemistry, physics, biology and mathematics. Symmetry may be the result of atomic arrangement as in crystals, or due to the growth pattern dictated by genes in plants and animals. In optics,

light can be directed in a symmetrical way, as in the pinhole camera, while in architectural structures, balance and proportion are basic requirements, not only for aesthetic purposes, but to counteract precisely the forces of gravity. Symmetry operations are found in logic and art as well. If you wish to pursue this fascinating subject further, here are a number of references that should be helpful:

"Can Time Go Backward?", by Martin Gardner, *Scientific American,* Jan. 1967.

Crystals, Charles Bunn, Academic Press, Inc., New York.

Crystals and Crystal Growing, Alan Holden and Phylis Singer, Doubleday & Co., New York.

Graphic Work of M. C. Escher, M. C. Escher, Duell, New York.

Introduction to Geometry, Coxeter, John Wiley and Son, Inc., New York.

Symmetry, Hermann Weyl, Princeton University Press, Princeton, New Jersey.

Symmetry in Chemistry, H. H. Jaffe and Milton Orchin, John Wiley & Sons, Inc., New York.

The individual packets of salt were

contributed by the Diamond Crystal Salt Co., Wilmington, Mass., and the zinnia seeds were packaged especially for this unit by the Asgrow-Mandeville Company, Cambridge, New York.

Appreciation is expressed to Dr. Morris Newman, National Bureau of Standards, for reviewing this booklet.

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